AP Calculus Name_____ Pd._____

Motion

1. The height (in meters) of a projectile shot vertically upward from a point 2 meters above ground level with an initial velocity of 24.5 m/s is $h = 2 + 24.5t - 4.9t^2$ after t seconds.

a.) Find the velocity after 2 s and after 4s.

$v(t) = 24.5 - 9.8t$ $v(2) = 4.9\ m/s$

$v(4) = -14.7\ m/s$

b.) When does the projectile reach its maximum height?

$0 = 24.5 - 9.8t$ $t = 2.5\ s$

c.) What is the maximum height?

$h(2.5) = 32.625\ ft$

d.) When does it hit the ground?

$t = 5$ sec

e.) With what velocity does it hit the ground?

$v(5) = 24.5 - 9.8(5)$

$= -24.5\ ft/sec$

2. If a ball is thrown vertically upward with a velocity of 80ft/s, then its height after t seconds is $s = 80t - 16t^2$.

a.) What is the maximum height reached by the ball?

$s'(t) = v(t) = 80 - 32t$ $s(2.5) = 100\ ft$

$0 = 80 - 32t$ $t = 2.5$

b.) What is the velocity of the ball when it is 96 ft. above the ground on its way up? On its way down?

$96 = 80t - 16t^2$

$\therefore 16t^2 - 80t + 96 = 0$

$16(t^2 - 5t + 6) = 0$

$16(t-3)(t-2) = 0$

$t = 3$ $t = 2$

$v(2) = s'(2) = 16\ ft/sec$ ↑

$v(3) = s'(3) = -16\ ft/sec$ ↓

3. If a rock is thrown vertically upward from the surface of Mars with velocity 15m/s, it height after t seconds is $h = 15t - 1.86t^2$.

a.) What is the velocity of the rock after 2 s?

$v(t) = h' = 15 - 3.72t$ $v(2) = 7.56\ m/s$

b.) What is the velocity of the rock when its height is 25m on its way up? On its way down?

$25 = 15t - 1.86t^2$

$\therefore 1.86t^2 - 15t + 25 = 0$

graph and find x-intercepts

$t = 2.353$ $t = 5.711$

$v(2.353) = 6.245\ m/s$ ↑

$v(5.711) = -6.245\ m/s$ ↓

4. A particle moves with position function: $s = t^4 - 4t^3 - 20t^2 + 20t$ $(t \geq 0)$

a.) At what time does the particle have a velocity of 20m/s?

$V(t) = s'(t) = 4t^3 - 12t^2 - 40t + 20$

$20 = 4t^3 - 12t^2 - 40t + 20$

$0 = 4t^3 - 12t^2 - 40t$

$0 = 4t(t^2 - 3t - 10)$

$0 = 4t(t-5)(t+2)$

$t = 0$ sec $\quad t = 5$ sec ~~$t = -2$~~

b.) At what time is the acceleration 0? What I the significance of this value t?

$a(t) = v'(t) = 12t^2 - 24t - 40$

$0 = 12t^2 - 24t - 40$

graph and find x-intercepts $t \geq 0$

$t = 3.0817$ sec

time at which the particle is not accelerating

* maximum or minimum velocity *

5. The cost, in dollars, of producing x yards of a certain fabric is $C(x) = 1200 + 12x - 0.1x^2 + 0.0005x^3$

Marginal cost = derivative of total cost

a.) Find the marginal cost function.

$C'(x) = 12 - .2x + .0015x^2$

b.) Find C'(200) and explain its meaning. What does it predict?

$C'(200) = 32$ \$/yard

the marginal cost of producing 200 yards of fabric is \$32/yard

c.) Compare C'(200) with the cost of manufacturing the 201st yard of fabric.

$C(201) = \$3632.20$

$C(200) = \$3600$

Cost of producing 201st yard is

$3632.20 - 3600 = \$32.20$

marginal cost

6. The cost function from production of a commodity is $C(x) = 339 + 25x - 0.09x^2 + 0.0004x^3$

a.) Find and interpret C'(100).

$C'(x) = 25 - .18x + .0012x^2$

$C'(100) = \$19$

marginal cost of producing 100th item is \$19 per item

b.) Compare C'(100) with the cost of producing the 101st item.

$C(101) = \$2358.03$

$C(100) = \$2339$

$\$2358.03 - 2339 = \$19.03$

marginal cost

Cost of 101st item