Calculus

Implicit Differentiation

Differentiate each.

1. $x^3 + y^3 = 6xy$

$3x^2 + 3y^2y' = 6xy' + 6y$

$3y^2y' - 6xy' = 6y - 3x^2$

$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \boxed{\frac{2y - x^2}{y^2 - 2x}}$

2. $x^3 + x^2y + 4y^2 = 6$

$3x^2 + x^2y' + 2xy + 8yy' = 0$

$x^2y' + 8yy' = -3x^2 - 2xy$

$\boxed{y' = \frac{-3x^2 - 2xy}{x^2 + 8y}}$

3. $x^2y + xy^2 = 3x$

$x^2y' + 2xy + x \cdot 2yy' + y^2 = 3$

$x^2y' + 2xyy' = 3 - 2xy - y^2$

$\boxed{y' = \frac{3 - 2xy - y^2}{x^2 + 2xy}}$

4. $4\cos x \sin y = 1$

$4\cos x \cdot \cos y \cdot y' + \sin y \cdot (-4\sin x) = 0$

$4\cos x \cos y \cdot y' - 4\sin x \sin y = 0$

$y' = \frac{4\sin x \sin y}{4\cos x \cos y} = \boxed{\tan x \tan y}$

5. $x\cos y + y\cos x = 1$

$x(-\sin y \cdot y') + \cos y + y(-\sin x) + \cos x y' = 0$

$-x\sin y \cdot y' + \cos x y' = -\cos y + y\sin x$

$y' = \boxed{\frac{-\cos y + y\sin x}{-x\sin y + \cos x}}$

6. $y = \cos(xy)$

$y' = -\sin(xy)(xy' + y)$

$y' = -xy'\sin(xy) - y\sin(xy)$

$y' + xy'\sin(xy) = -y\sin(xy)$

$y' = \boxed{\frac{-y\sin(xy)}{1 + x\sin(xy)}}$

Expand!! Implicit Differen… Remember D-N… (AT)] = $\frac{1}{f(x)}$

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Name________________________

7. Find $\frac{d^2y}{dx^2}$ for $x^2+y^2=25$

$2x+2yy'=0$

$y'=\frac{-2x}{2y}=\frac{-x}{y}$

$y''=\frac{y(-1)-(-x)y'}{y^2}=\frac{-y+xy'}{y^2}$

$=\frac{-y+x\left(\frac{-x}{y}\right)}{y^2}=\frac{-y-\frac{x^2}{y}}{y^2}\cdot\frac{1}{y^2}$

$=-\frac{1}{y}-\frac{x^2}{y^3}=\frac{-y^2-x^2}{y^3}$

8. Find an equation of the tangent line to the curve $y^2=x^3(2-x)$ at the point (1,1).

$2yy'=x^3(-1)+(2-x)\cdot 3x^2$

$2yy'=-x^3+6x^2-3x^3$

$2yy'=-4x^3+6x^2$

$y'=\frac{-4x^3+6x^2}{2y}=\frac{-2x^3+3x^2}{y}\Big|_{(1,1)}=\frac{-2+3}{1}=1$

$\boxed{y-1=1(x-1)}$

9. Find all point on the curve $x^2y^2+xy=2$ where the slope of the tangent line is -1.

$x^2\cdot 2yy'+y^2\cdot 2x+x\cdot y'+y=0$

$2x^2yy'+2xy^2+xy'+y=0$

$2x^2yy'+xy'=-2xy^2-y$

$y'=\frac{-2xy^2-y}{2x^2y+x}=\frac{-y\,\cancel{(2xy+1)}}{x\,\cancel{(2xy+1)}}=\frac{-y}{x}$

$-1=\frac{-y}{x}$

$-x=-y$

$x=y$ ← sub into original equation

$x^2y^2+xy=2$

$y^2\cdot y^2+y^2=2$

$y^4+y^2=2$

$y^4+y^2-2=0$

$(y^2-1)(y^2+2)=0$

$y^2=1$ $\quad$ $\cancel{y^2=-2}$

$y=\pm 1$

$\boxed{(-1,-1)\ (1,1)}$

10. Find the equations of both the tangent lines to the ellipse $x^2+4y^2=36$ that pass through the point $(12,3)$.

$2x+8yy'=0$

$y'=\frac{-2x}{8y}$

$y'=\frac{-x}{4y}$