AP Calculus

Name_____ Pd.____

Implicit Differentiation

1-10: Find dy/dx by implicit differentiation.

1. $x^3 + y^3 = 1$

$$y' = \frac{-x^2}{y^2}$$

2. $x^2 + xy - y^2 = 4$

$$y' = \frac{-2x - y}{x - 2y}$$

3. $2x^3 + x^2y - xy^3 = 2$

$$y' = \frac{-6x^2 - 2xy + y^3}{x^2 - 3xy^2}$$

4. $xe^y = x - y$

$$y' = \frac{1 - e^y}{xe^y + 1}$$

5. $y\cos x = x^2 + y^2$

$$y' = \frac{2x + y\sin x}{\cos x - 2y}$$

6. $\cos(xy) = 1 + \sin y$

$$y' = \frac{y\sin(xy)}{-x\sin(xy) - \cos y}$$

7. $4\cos x \sin y = 1$

$$y' = \frac{\sin x \sin y}{\cos x \cos y} = \tan x \tan y$$

8. $e^y \sin x = x + xy$

$$y' = \frac{1 + y - e^y \cos x}{\sin x e^y - x}$$

9. $e^{\frac{x}{y}} = x - y$

$$y' = \frac{y^2 - ye^{\frac{x}{y}}}{y^2 - xe^{\frac{x}{y}}}$$

10. $\sqrt{x+y} = 1 + x^2y^2$

$$y' = \frac{4xy^2\sqrt{x+y} - 1}{1 - 4x^2y\sqrt{x+y}}$$

11-12: Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

11. $\sin 2x = x\cos 2y, \quad \left(\frac{\pi}{2}, \frac{\pi}{4}\right)$

$$y' = \frac{2\cos(2x) - \cos(2y)}{-2x\sin(2y)}\Big|_{(\frac{\pi}{2},\frac{\pi}{4})} = \frac{2}{\pi}$$

$$y - \frac{\pi}{4} = \frac{2}{\pi}\left(x - \frac{\pi}{2}\right)$$

12. $x^2 + 2xy - y^2 + x = 2, \quad (1,2)$

$$y' = \frac{-1 - 2x - 2y}{2x - 2y}\Big|_{(1,2)} = \frac{7}{2}$$

$$y - 2 = \frac{7}{2}(x - 1)$$

13-14: Find y'' by implicit differentiation.

13. $9x^2 + y^2 = 9$

$$y' = \frac{-9x}{y}$$

$$y'' = \frac{-9y - \frac{81x^2}{y}}{y^2} = \frac{-9}{y} - \frac{81x^2}{y^3}$$

14. $x^4 + y^4 = a^4 \quad a^4 = \text{constant}$

$$y' = \frac{-x^3}{y^3}$$

$$y'' = \frac{-3x^2y^3 - \frac{3x^6}{y}}{y^6} = \frac{-3x^2y^3}{y^6} - \frac{3x^6}{y^7}$$